PHYS 709: Advanced Mechanics I

Homework #5 - **Due Monday, December 2nd, 10 am**

Name:

Here, GS = Goldstein, TM = Thornton and Marion, FW = Fetter and Walecka. As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.

1. **GS Ch3 Q14**
   Consider an attractive $1/r$ potential and show that:
   (a) for circular and parabolic orbits having the same angular momentum, the perihelion distance of the parabola is one-half the radius of the circle.
   (b) that in the same central force as in part (a) the speed of a particle at any point in a parabolic orbit is $\sqrt{2}$ times the speed in a circular orbit passing through the same point.

2. **FW 1.14** The cross section to strike the nuclear surface is of interest in discussing nuclear reactions during heavy-ion scattering. By integrating over the appropriate impact parameters, show that the cross section to strike a nucleus of radius R in Rutherford scattering is given by:
$$\sigma_r = \pi R^2(1 - V_c/E)$$
   where $V_c = zZe^2/R$ is the repulsive Coulomb barrier at the nuclear surface and it is assumed that $E \geq V_c > 0$.

3. **FW 1.6**
   A uniform beam of particles with energy E is scattered by a repulsive central potential $V(r) = \gamma/r^2$.
   Derive the differential cross section:
$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{\gamma\pi^2}{E\sin(\theta)}\frac{\pi - \theta}{\theta^2(2\pi - \theta)^2}$$
   Sketch carefully the angular dependence. Discuss the total cross section. What happens if the potential is attractive, that is, $\gamma > 0$?

4. **Computing Task #5**
   Create a Jupyter notebook and use the differential cross section from the previous question (3. FW 1.6). Plot the differential cross section as a function of θ in the range $0 \leq \theta \leq \pi$. Justify values of γ and E you choose. It is possible doing this Computing Task first may help your understanding of Q3 better.

5. **GS Ch4 Q4**
Show that if $\mathbf{A}$ is a real 3×3 anti-symmetric matrix, then the matrices $\mathbb{1} \pm \mathbf{A}$ are non-singular (invertible; determinant is not zero), and the matrix $\mathbf{B} = (\mathbb{1} + \mathbf{A})(\mathbb{1} - \mathbf{A})^{-1}$ is orthogonal.

6. **GS Ch4 Q14**

   (a) Verify that the [Levi-Cevita] permutation symbol satisfies the following identity in terms of Kronecker delta symbols:

   $$\epsilon_{ijp}\epsilon_{rmp} = \delta_{ir}\delta_{jm} - \delta_{im}\delta_{jr}$$

   (b) Show that

   $$\epsilon_{ijp}\epsilon_{ijk} = 2\delta_{pk}$$

7. **GS Ch4 Q15**
Show that the components of the angular velocity along the space set of axes are given in terms of the Euler angles by:

$$\begin{aligned} \omega_x &= \dot{\theta}\cos\phi + \dot{\psi}\sin\theta\sin\phi \\ \omega_y &= \dot{\theta}\sin\phi - \dot{\psi}\sin\theta\cos\phi \\ \omega_z &= \dot{\psi}\cos\theta + \dot{\phi} \end{aligned}$$

8. **GS 4.24**
A wagon wheel with spokes is mounted on a vertical axis so it is free to rotate in the horizontal plane. Thew wheel is rotating with an angular speed of $\omega = 3.0$ radians/s. A bug crawls out on one of the spokes of the wheel with a velocity of 0.5 cm/s holding on to the spoke with a coefficient of friction $\mu = 0.30$. How far can the bug crawl along the spoke before it starts to slip?

9. **FW 2.4** A particle is projected vertically upward from the surface of the rotating Earth at colatitude (polar angle) θ. It rises to a height h ($\ll R_E$) and then falls back to the surface. Show that it strikes the ground at a distance:

$$\frac{8}{3}\omega\sin\theta(2h^3/g)^{1/2}$$

to the west of the initial position.
What is its westward displacement at its maximum height?