PHYS 709: Advanced Mechanics I

Homework #4 - **Due Monday, October 21, 10 am**

Name:

Here, GS = Goldstein, TM = Thornton and Marion, FW = Fetter and Walecka, and HF = Hand and Finch. As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.
Note that a least one of these questions you've gone over together in class so they should be a nice review of what you remember.

1. **FW 4.1**
   A thin hoop of radius R and mass M oscillates in its own plane with one point of the hoop fixed. Attached to the hoop is a point mass M constrained to move without friction along the hoop. The system is in a uniform gravitational field $\mathbf{g}$. Consider only small oscillations.

   (a) Show that the normal-mode frequencies are

   $$\omega_1 = \frac{1}{2}\left(\frac{2g}{R}\right)^{1/2}$$

   $$\omega_2 = \left(\frac{2g}{R}\right)^{1/2}$$

   (b) Find the normal-mode eigenvectors. Sketch the motion.

   (c) Construct the modal matrix.

   (d) Find the normal coordinates and show that they diagonalize the lagrangian.

2. **FW 4.2**
   Consider the longitudinal oscillations, i.e. along the axis, of the mechanical system in Fig. 24.1 (FW) assuming only two masses and three springs. Work from first principles.

   (a) Find the Langrangian and Langrange's equations

   (b) What are the normal-mode frequencies and eigenvectors? Describe the motions.

   (c) Construct the modal matrix and normal coordinates, and write the lagrangian in diagonal form.

   (d) Suppose the mass on the left is initially displaced from equilibrium a distance α to the right. Compute the subsequent motion.

   (e) Compare the treatment with the general results in Sec. 24 (FW) for $N = 2$.

3. **FW 4.8**
   Four massless rods of length L are hinged together at their ends to form a rhombus. A particle of mass M is attached at each joint. The opposite corners of the rhombus are joined by springs, each with a spring constant k. In the equilibrium (square) configuration, the springs are unstretched. The motions is confined to a plane, and the particles move only along the diagonals of the rhombus. Introduce suitable generalized coordinates and find the lagrangian of the system. Deduce the equations of motion and find the frequency of small oscillations about the equilibrium configuration.

4. **GS 6.18**
   A particle in an isotropic three-dimensional harmonic oscillator potential has a natural frequency of ω_0. Assume the particle is charged and that crossed static electric and magnetic fields are applied. Find the vibration frequencies with these electromagnetic fields present. Discuss the results for the limits of strong and weak fields.