PHYS 709: Advanced Mechanics I

Homework #3 - **Due Wednesday, October 02, 10 am**

Name:

Here, GS = Goldstein, TM = Thornton and Marion, FW = Fetter and Walecka, and HF = Hand and Finch. As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.

1. **FW 6.1: Hamiltonian Dynamics**
   A particle moves in a central potential $V(r)$. Use three-dimensional spherical polar coordinates (r, θ, ϕ) to obtain the Hamiltonian. Derive the equations of motion.

2. **TM modified: Hamiltonian EOM**
   Consider a particle of mass m which is constrained to a semi-circle of radius R of equation $x^2 + z2 = R^2$, where $z < 0$, under the action of the gravity. Find the Hamiltonian and subsequently the equations of motion.

3. **Poisson 3.6.1 - Cycloid Wire (or Sliding Bead revisited)**
   A bead of mass m slides on a frictionless wire that is shaped in the form of a cycloid. This is described by the parametric equations

   $$x = a(\theta - \sin\theta) \qquad y = a(1 + \cos\theta)$$

   where a is a constant and the parameter θ ranges through the interval $0 \leq \theta \leq 2\pi$. The bead is subjected to gravity, and it oscillates back and forth on the wire. [Hint: You can use the result from the previous homework to find the Hamiltonian.]

   (a) Using θ as a generalized coordinate, calculate the bead's Hamiltonian.

   (b) Obtain Hamilton's canonical equations of motion for the bead.

   More questions on next page....

4. **GS Ch8 Q26**
A particle of mass m can move in one dimension under the influence of two springs connected to fixed points a distance a apart (see diagram in textbook). The springs obey Hooke's law and have zero unstretched lengths and force constants k_1 and k_2.

   (a) Using the position of the particle from the fixed point as the generalized coordinate, find the Lagrangian for the system. What is the corresponding Hamiltonian? Is energy conserved? Is the Hamiltonian conserved?

   (b) Introduce a new coordinate Q defined by:

$$Q = q - b\sin\omega t$$
$$b = \frac{k_2 a}{k_1 + k_2}$$

5. **HF 6.14: Poisson Brackets**
Note: Poisson brackets of constants of the motion can generate new constants of the motion.
Consider the uniform motion of a free particle of mass m. The Hamiltonian is a constant of the motion, and so is the quantity F defined as: $F(x, p, t) \equiv x - \frac{pt}{m}$.

   (a) Compare [H, F] with $\frac{\partial F}{\partial t}$.

   (b) Prove that the Poisson bracket of two constants of the motion is itself a constant of the motion, even if the constants $F(x, p, t)$ and $G(x, p, t)$ depend explicitly on the time (Part (a) is one example of this.)

   (c) Show *in general* that if the Hamiltonian and quantity F are constants of the motion then $\frac{\partial F}{\partial t}$ is a constant of the motion also.