PHYS 709: Advanced Mechanics I

Homework #2 - **Due Monday, September 16, 10 am**

Name:

Here, GS = Goldstein, and TM = Thornton and Marion. As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.

"I regard as quite useless the reading of large treatises of pure analysis; too large a number of methods pass at once before the eyes. It is in the works of application that one must study them;.."
– Joseph-Louis Lagrange

1. **Poisson 2.3 - Sliding Bead**
   A bead of mass m slides on a frictionless wire that is shaped in the form of a cycloid. This is described by the parametric equations

   $$x = a(\theta - \sin\theta) \qquad y = a(1 + \cos\theta)$$

   where a is a constant and the parameter θ ranges through the interval $0 \leq \theta \leq 2\pi$. The bead is subjected to gravity, and it oscillates back and forth on the wire.

   (a) Using θ as a generalized coordinate, calculate the bead's Lagrangian.

   (b) Show that the equation of motion for the bead is

   $$2(1 - \cos\theta)\ddot{\theta} + \sin\theta\,\dot{\theta}^2 - \frac{g}{a}\sin\theta = 0$$

   (c) Show that the transformation $u = \cos\left(\frac{1}{2}\theta\right)$ brings the equation to the much simpler form

   $$\ddot{u} + \omega^2 u = 0$$

   and find an expression for ω.

   (d) What is the period of the bead's oscillations?

2. **Poisson 2.9 – Electromagnetic Potentials**
   A particle of mass m and electric charge q moves in the presence of a vector potential

   $$\mathbf{A} = \frac{1}{2}B_0\left(-y\hat{\mathbf{x}} + x\hat{\mathbf{y}}\right)$$

   where B_0 is a constant.

   (a) What is the magnetic field $\mathbf{B}$?

(b) What is the particle's Lagrangian?

(c) What are the particle's equations of motion?

(d) What is the general solution to these equations? Describe how the particle moves in this magnetic field?

3. **GS Ch2 Q14**
A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown in the figure. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange multipliers to find the point at which the hoop falls off the cylinder,

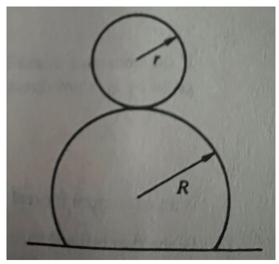

Figure 1: GS Ch2 Q14 setup

4. **GS Ch2 Q18**
A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω. Obtain the Lagrange equations of motion assuming the only external forces arise from gravity. What are the constants of motion?

5. **GS Ch2 Q19**
A particle moves without friction in a conservative field of force produced by various mass distributions. In each instance, the force generated by a volume element of the distribution is derived from a potential that is proportional to the mass of the volume element and is a function only of the scalar distance from the volume element. For the following fixed, homogeneous mass distributions, state the conserved quantities in the motion of the particle:

(a) The mass is uniformly distributed in the plane $z = 0$

(b) The mass is uniformly distributed in the half-plane $z = 0, y > 0$

(c) The mass is uniformly distributed in a circular cylinder of **infinite** length, with axis along the z axis

(d) The mass is uniformly distributed in a circular cylinder of **finite** length, with axis along the z axis

(e) The mass is uniformly distributed in a right cylinder of elliptical cross-section and infinite length, with axis along the z axis

(f) The mass is uniformly distributed in a dumbbell whose axis is oriented along the z axis

(g) The mass is in the form of a uniform wire wound in the geometry of an infinite helical solenoid, with axis along the z axis