PHYS 709: Advanced Mechanics I

Homework #2 - **Due Friday, September 06, 10 am**

Name:

Here, GS = Goldstein, and TM = Thornton and Marion. As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.

1. **Conservative forces?**
   We discussed how work done in moving from one point to another is path-independent; work vanishes for a closed path. This is a property of conservative forces. Relating this to Stokes' theorem concludes that any conservative force must have a vanishing curl:

   $$\vec{\nabla} \times \vec{F} = 0 \tag{1}$$

   For the following, calculate the curl of the given vector field and comment on whether the force is conservative or not. If it is conservative, find the potential energy U($\vec{r}$) associated with the force.

   (a) $F_x = ayz + bx + c$, $F_y = axz + bz$, $F_z = axy + by$.

   (b) $F_x = -ze^{-x}$, $F_y = \ln Z$, $F_z = -e^{-x} + y/z$. Consider only for z > 0.

2. **GS 1**
   Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy:

   $$\frac{dT}{dt} = \vec{F} \cdot \vec{v} \tag{2}$$

   while if the mass varies with time the corresponding equation is

   $$\frac{d(mT)}{dt} = \vec{F} \cdot \vec{p} \tag{3}$$

3. **TM 7.1**
   Find a suitable set of generalized coordinates for a point particle moving on the surface of a hemisphere of radius R whose center is at the origin.

4. **TM 7.3 modified**
   Consider the case of projectile motion under gravity in two dimensions: a cannon in an x-y plane firing a cannonball with some initial velocity $\mathbf{v}_0$ at some angle θ from the horizontal x-axis.

   (a) Find the equations of motion in both Cartesian and polar coordinates.

(b) Comment and discuss which of the two coordinate systems one should choose as generalized coordinates.

5. **GS 14**
   Two points of mass m are joined by a rigid weightless rod of length l, the center of which is constrained to move on a circle of radius a. Express the kinetic energy in terms of generalized coordinates.

6. **GS 19**
   Obtain the Lagrange equations of motion for a spherical pendulum, i.e. a mass point suspended by a rigid weightless rod.